PADHO TO AISE

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UGC NET PAPER-1

FREE MATERIALS

CHAPTER- 5

Mathematical Reasoning and Aptitude

18. Types of Reasoning

REASONING (INCLUDING MATHEMATICS)

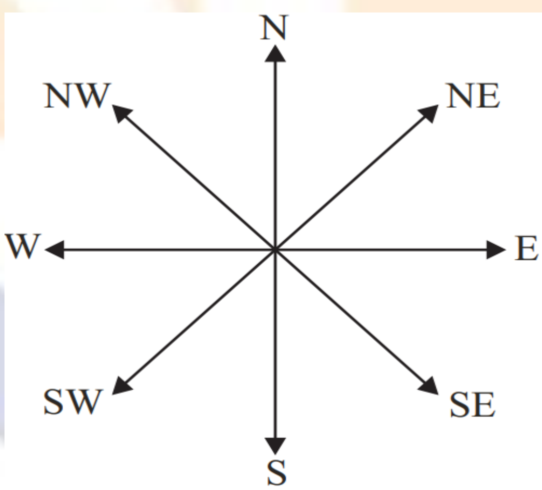
Reasoning is fundamental to knowing and doing mathematics. In mathematics, reasoning involves drawing logical conclusions based on evidence or stated assumptions. In common language, to reason is to think, understand and form judgements logically or to find a solution to a problem by considering the possible options. Aptitude is a natural ability or propensity to learn. Sense making may be considered as developing understanding of a situation, context or concept by connecting it with existing knowledge or previous experience. Reasoning and sense making are closely interrelated and are the foundation for a solid preparation in mathematics. In this unit, apart from reasoning, we will discuss mathematical aptitude also that has very important in NET Examination.

There are two types of Reasoning tests - verbal and Non-verbal. The items in a verbal test are stated in language form such as Analogy test, series test, different class test, etc. Whereas, Nonverbal tests are made up of materials such as patterns, lines, drawings, pictures, paper cutting and the like. Non-verbal tests are also called matrices or abstract reasoning tests. As far as NET examination pattern is concerned mathematical reasoning covers mainly verbal and basic mathematical skills. Questions on series, analogy, classification, blood relation, direction sense, seating arrangement appear regularly in the exam.

It is also necessary to be familiar with simple, basic mathematical operations such as simplification, percentage, profit, loss and discount, Ratio and proportion, Average, time, speed and distance, time and work, etc.

DIRECTION SENSE TEST

In this part, the questions consist of a sort of direction puzzle and the candidates are required to ascertain the final direction or the distance between the two points. The test is meant to judge the candidate's ability to trace and follow the direction correctly. The adjoining diagram shows the four main directions (North N, South S, East E, and West W) and the four cardinal directions (North-East NE, North-West NW, South-East SE, and South-West SW). Candidates should memorize the same.



How to solve the problems

The easiest way of solving these problems is to draw a diagram as you read information about the problem and let the diagram reflect all the information given in the problem. To solve these types of problems, the student should know the directions properly without any confusion.

POINTS TO REMEMBER

• At the time of sunrise if a man stands facing the east, his shadow will be towards west, i.e., behind him.

• At the time of sunset, the shadow of an object is always to the east.

• If a man stands facing the North, at the time of sunrise his shadow will be towards his left and at the time of sunset it will be towards his right.

• At 12:00 noon, the rays of the sun are vertically downward and hence there will be no shadow.

• The shortest distance from a particular point after travelling a distance of x metres in the horizontal direction and a distance of y metres in the vertical direction is equal to $\sqrt{x^2 + y^2}$

RANKING & NUMBER TEST, TIME SEQUENCE

I. Rank of a person in a queue

• Position of person from upward

= [Total no. of persons – position of person from down] + 1.

• Position of person from downward

= [Total no. of persons – position of person from up] + 1.

• Position of person from right

= [Total no. of persons – position of person from left] + 1.

• Position of person from left

= [Total no. of persons – position of person from right] + 1.

CLOCK & CALENDAR

• The solar year consists of 365 days, 5 hrs 48 minutes, 48 seconds.

In 47 BC, Julius Caesar arranged a calendar known as the Julian calendar in which a year was taken as days and in order to get rid of the $365\frac{1}{4}$ odd quarter of a day, an extra day was added once in every fourth year and this was called as a leap year.

• In an ordinary year,

1 year = 365 days = 52 weeks + 1 day

In a leap year,

1 year = 366 days = 52 weeks + 2 days

Note : First January 1 A.D. was Monday.

To find a particular day corresponding to a particular date, the number of odd days upto that date should be computed and if no. of odd days is

0 odd day stands for Sunday

1 odd day stands for Monday

2 odd day stands for Tuesday

3 odd day stands for Wednesday

4 odd day stands for Thursday

5 odd day stands for Friday

6 odd day stands for Saturday

• A clock has two hands : Hour hand and Minute hand.The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.

• The clock has 12 hours numbered from 1 to 12. Also, the clock is divided into 60 equal minute divisions. Therefore, each hour number is separated by five-minute divisions. Therefore,

• Two One-minute divisions = $\frac{360}{60}$= 6° apart i.e., in one minute, the minute hand moves 6°.

• Two One-hour divisions = 6° × 5 = 30° apart i.e., in one hour, the hour hand moves 30°. Also, in one minute, the hour hand moves = $\frac{30°}{60°}$ = $\frac{1°}{2°}$

• Since in one minute, the minute hand moves 6° and hour hand moves $\frac{1°}{2°}$, therefore, in one minute, the minute hand gains $5\frac{1°}{2°}$ over the hour hand.

• In one hour, the minute hand gains $5\frac{1^\circ}{2^\circ} \times 60 = 330^\circ$ over the hour hand i.e. the minute hand gains 55-minute divisions over the hour hand.

• The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on the same side (clockwise or anticlockwise) of the H.H. Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

• When both hands are 15-minute spaces apart, they are at a right angle. When they are 30-minute spaces apart, they point in opposite directions. The hands are in the same straight line when they are coincident or opposite to each other.

• In every hour, both the hands coincide once.

– In a day, the hands are coinciding 22 times.

– In every 12 hours, the hands of a clock coincide 11 times.

– In every 12 hours, the hands of a clock are in opposite direction 11 times.

– In every 12 hours, the hands of clock are at right angles 22 times.

– In every hour, the two hands are at right angles 2 times.

– In every hour, the two hands are in opposite directions once.

– In a day, the two hands are at right angles 44 times

• If both the hands coincide, then they will again coincide after $65\frac{5}{11}$ minutes. i.e., in a correct clock, both hands coincide at an interval of $65\frac{5}{11}$ minutes.

• If the two hands coincide in time less than $65\frac{5}{11}$ minutes, then the clock is running too fast and if the two hands coincide in time more than $65\frac{5}{11}$ minutes, then the clock is running too slowly.

• If a clock indicates 6: 10, when the correct time is 6: 00, it is said to be 10 minutes too fast and if it indicates 5: 50 when the correct time is 6: 00, it is said to be 10 minutes too slow.

• Also, if both hands coincide at an interval of x minutes and x <$65\frac{5}{11}$ then total time gained = $\left(\frac{65\frac{5}{11}-x}{x}\right)$ and clock is said to be 'fast'.

• If both hands coincide at an interval of x minutes and x <$65\frac{5}{11}$, hen total time lost = $\left(\frac{x-65\frac{5}{11}}{11}\right)$ minutes and clock is said to be 'slow'.

SITTING ARRANGEMENT & PUZZLE

In this section, generally the questions are asked from Ordering or Ranking, Scheduling, Scattering or Distributing. In these types of questions, we have to analyse the given information and condense it in a suitable form to answer the questions. Though there exist no set formulae to solve these kinds of problems, yet a systematic approach can help to solve questions. As far as possible, it should be tried to tabulate the data, as it helps us to condense the information and reach to conclusions.

MATHEMATICAL OPERATION AND ARITHMETICAL REASONING

MATHEMATICAL OPERATION

In such type of questions some relationships are shown with the help of certain symbols/notations and/or mathematical signs. Each symbol or sign is defined clearly in the question statement itself and we have to solve the questions accordingly.

For example,

Suppose the triangle (D) means addition. We know that triangle is a plane figure but here it has been assigned the value of addition (+). Thus.

$$3 \triangle 5 \Rightarrow 3 + 5 = 8$$

To work out such questions substitute the assigned/implied meanings of the symbol or sign and proceed accordingly. Do not consider its real meaning and follow the BODMAS rule.

MATHEMATICAL STATEMENTS

Mathematical Statements are the combination of mathematical signs, symbols, or letters which are based on certain mathematical rules. **For example:**

‘P < Q’ means ‘P is smaller than Q’.

‘P ≤ Q’ means ‘P is either smaller than or equal to Q’.

‘P > Q’ means ‘P is greater than Q’.

‘P ≥ Q’ means ‘P is either greater than or equal to Q’.

‘P = Q’ means ‘P is equal to Q’.

Conclusions: conclusion is the last main division of a discourse, usually containing a summing up of the points of given statement.

For example:

A > B, C > A

Conclusions:

I. C > B

II. C = B

After combining the above statements, we find that C > A > B Here C is greater than both of A and B but C is not equal to B. Hence conclusion II is incorrect.

19. Numerical Series, Letter Series, Codes and Relationship

NUMERICAL SERIES

Number Series tests are a type of numerical aptitude test which require you to find the missing or wrong number in a sequence. This missing or wrong number may be at the beginning or middle or at the end of sequence. The only thing to understand for solving these questions is the pattern on which a number series is written. A number series can be framed by using various methods. Therefore, it is advisable for the students to practice as many questions as possible.

ALPHABET SERIES

There are 26 alphabets in the english alphabet series. If the alphabets are asked to be counted from the left side, we start counting from the letter A, and when asked to count from right we count from Z.

A	B	C	D	E	F	G	H	I
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
J	K	L	M	N	O	P	Q	R
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
S	T	U	V	W	X	Y	Z	
(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26).	

1. Keep the order of letters with their respective numbers i.e. A = 1, ... Z = 26 and vice-versa i.e. Z = 1 .... A = 26.

2. When the counting is circular, after Z, the cycle will continue from A.

3. Sometimes letters are omitted in sequence e.g., ABCD, FGH, here E and I in the sequence are omitted.

4. Sometimes, equal number of letters are omitted e.g. in BEH, two consecutive letters are omitted.

5. One following the letter and other preceding it may be interchanged, e.g., in BCD, D follows C and B precedes C.

6. Sometimes, numbers are mixed with letters, the number may refer to the position of the letters in the alphabet.

7. Sometimes, there may be repetition of letters in a set order e.g. in abcc, bcdd, one letter is repeated twice, the next set could be cdee.

CODING & DECODING

A code is a method of sending a secret message between two parties that cannot be deciphered by a third party. However Coding is done according to a certain pattern in the mind of the sender. Therefore, its meaning can be deciphered by a third person, only if he carefully studies this pattern. This process is called 'Decoding'. You will be required to find a word by analysing the given code or forming the code for a new word.

Following are the general patterns for coding - Decoding-

(i) Words could be written in reverse order.

(ii) ±1, ±2, ±3......should be considered while decoding. So +1, means that B would be written in place of A and so on, –1 means A would be written in place of B and so on.

(iii) Sometimes, a successive increasing or decreasing pattern can also be followed, i.e., first alphabet would be replaced by +1, Second alphabet by +2 and so on.

(iv) Oscillation pattern can also be followed, i.e. first alphabet would be replaced by +1, second alphabet would be replaced by –1 and so on.

(v) Also, sometime the given code contains all the alphabets for the one given in question. In such a situation corresponding code should be replaced to curive at the answer.

ANALOGY

Analogy literally means 'similarity' or having similar features. Questions on analogy test the ability of a candidate to understand the relationship between two given objects or words or numbers that are asked in the question. These types of questions cover all types of relationships that one can think of, there are many ways of establishing a relationship.

CLASSIFICATION

Classification is a process of grouping various objects on the basis of their common properties. Classification, therefore, helps to make a homogeneous group from heterogeneous items. Questions on classification can be asked in any of the following form.

BLOOD RELATION

Questions based on blood relationships are very common. In these type of questions, a roundabout description is given in the form of certain small relationships and direct relationship between the persons concerned is to be deciphered. Questions based on Blood Relation may appear confusing but easier to answer if we break the questions into parts and analyze it with the help of a diagram. Representation of one generation to the next

1st generation – Grandfather, Grandmother

2nd Generation - Mother, Father, Uncle, Aunty

3rd Generation - Self, Sister, Brother & Brother/Sister-in-law

4rth Generation - Son, Daughter, Nephew, Niece

POINTS TO REMEMBER

Memorable facts about blood relations	
My mother's or father's son	my Brother.

My mother’s or father’s daughter	my Sister.
My mother’s or father’s father	my Grandfather.
My mother’s or father’s sister	my Aunt.
My mother’s or father’s brother	my Uncle.
My son’s wife	my daughter-in-law
My daughter’s husband	my Son-in-law.
My brother’s son	my Nephew
My brother’s daughter	my Niece.
My sister’s husband	my brother-in-law.
My brother’s wife	my Sister-in-law.
My husband’s or wife’s sister	my Sister-in-law.
My husband’s or wife’s brother	my Brother-in-law.
My uncle’s or aunt’s son or daughter	my Cousin
My wife’s father or husband’s father	my Father-in-law.
My wife’s mother or husband’s mother	my Mother-in-law.
My father’s wife	my Mother
My mother’s husband	my Father.
My son’s or daughter’s son	my Grandson

My son's or daughter's daughter	my Granddaughter.

VENN DIAGRAM

The best method of solving the problems based on inference or deduction is Venn diagram.

Venn diagram is a way representing sets pictorially. Various cases of Venn diagram

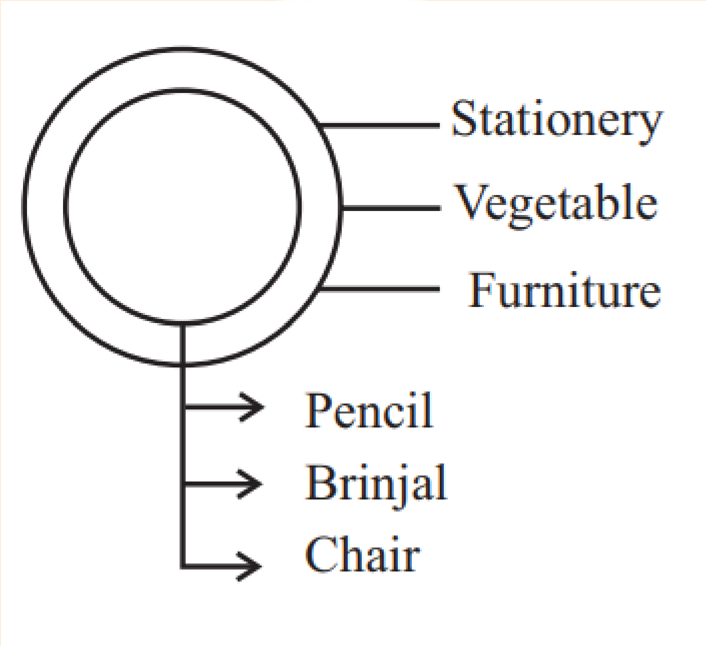
Case I:

An object is called a subset of another object, if the former is a part of the latter and such relation is shown by two concentric circles.

(i) Pencil, Stationery

(ii) Brinjal, Vegetable

(iii) Chair, Furniture It is very clear from the above relationship that one object is a part of the other, and hence all such relationships can be represented by the figure shown.



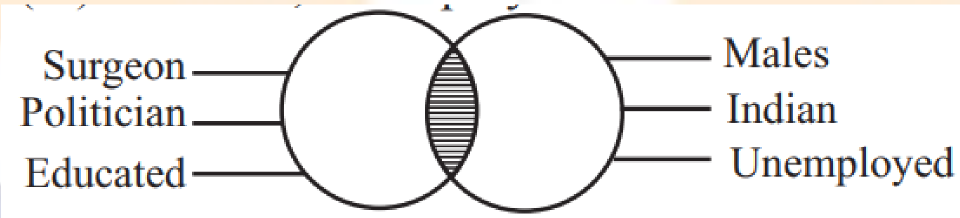
Case II:

An object is said to have an intersection with another object that share some things in common.

(i) Surgeon, Males

(ii) Politicians, Indian

(iii) Educated, Unemployed

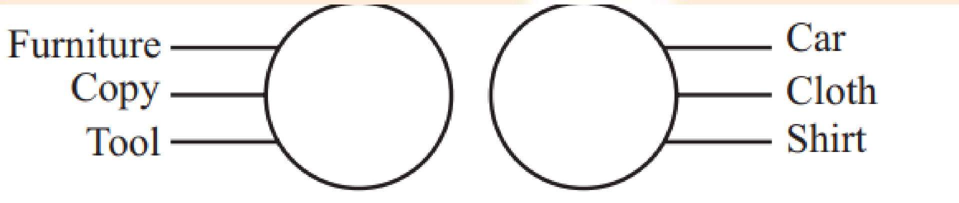


All the three relationships given above have something in common as some surgeons can be male and some female, some politicians may be Indian and some may belong to other countries, educated may be employed and unemployed as well and all the three relationships can be represented by the figure shown.

Case III :

Two objects are said to be disjoint when neither one is subset of another and nor do they share anything in common. In other words, totally unrelated objects fall under this type of relationship.

(i) Furniture, Car (ii) Copy, Cloth (iii) Tool, Shirt



It is clear from the above relationships that both the objects are unrelated to each other, and hence can be represented diagrammatically as shown in figure above.

From the above discussion we observe that representation of the relationship between two objects is not typical if students follow the above points. But representation of three objects diagrammatically pose slight problems before the students.

ANALYTICAL METHOD:

Try to understand these types of questions using analytical method. A statement always has a subject and a predicate:

All politicians are liars

(subject) (predicate)

Basically, there are four types of sentences.

A - type ⇒ All politicians are liars.

I-type ⇒ Some politicians are liars

O-type ⇒ Some politicians are not liars

E-type ⇒ No politicians are liars

Conclusions can be drawn by taking two of the above statements together. The rules of conclusion are :

A + A = A   A + E = E   I + A = I

I + A = I   E + A = O*   E + I = O*

Conclusion can only be drawn from the two statements if the predicate of the first statement is the subject of the second statement. The common term disappears in the conclusion and it consists of subject of the first statement and predicate of the second statement. For examples

A + A = A

(i) All boys are girls.

(ii) All girls are healthy

Conclusion: All boys are healthy.

A + E = E

(i) All boys are girls.

(ii) No girls are healthy

Conclusion: No boys are healthy

I + A = I

(i) Some boys are girls.

(ii) All girls are healthy

Conclusion: Some boys are healthy.

I + E = O

(i) Some boys are girls.

(ii) No girls are healthy

Conclusion: Some boys are not healthy.

E + A = O*

(i) No boys are girls.

(ii) All girls are healthy

Conclusion : Some healthy are not boys.

I + I = O*

(i) No boys are girls.

(ii) Some girls are healthy

Conclusion : Some healthy are not boys.

20. Mathematical Aptitude

SIMPLIFICATION

Algebraic expressions contain alphabetic symbols as well as numbers. When an algebraic expression is simplified, an equivalent expression is found that is simpler than the original. This usually means that the simplified expression is smaller than the original.

Bodmas Rule :

This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression. Here, 'B' stands for 'Bracket', 'O' for 'of', 'D' for 'Division, 'M' for 'Multiplication', 'A' for 'Addition', 'S' for 'Subtraction'. When an expression contains a vinculum (a horizontal line above an expression), before applying the 'BODMAS' rule, we simplify the expression under the vinculum. The next step is to evaluate all the expressions in the brackets. After removing the brackets, we must use the following operations strictly in the following order:

1. of

2. Division, Multiplication

3. Addtition, Subtraction

So, the order of precedence is:

V Vinculum

B Brackets [{(

O Of, Orders (i.e. Powers and Square Roots, etc.)

DM Division and Multiplication (left-to-right)

SURDS AND INDICES

Mixed surds: If one factor of a surd is a rational number and the other factor is an irrational number, then the surd is called a mixed surd.

Example: $2\sqrt{5}$, $-2\sqrt{3}$

Pure surds: If a surd has unity as its only rational factor, the other factor being an irrational number, then it is called a pure surd.

Examples: $\sqrt{3}, \sqrt{a}$

Since surds are irrational numbers, they can be added or subtracted as real numbers. Also a rational number can be added or subtracted from a surd. The result will be a real number.

Examples: $\sqrt{5}+3;\ 2-\sqrt{7};\ \sqrt{3}-2$

Addition and Subtraction of Surds :

Example: $5\sqrt{2}+20\sqrt{2}-3\sqrt{2}=22\sqrt{2}$

Example: $\sqrt{45}-3\sqrt{20}+4\sqrt{5}=\sqrt{5}$

Multiplying and Dividing Surds :

Surds can be multiplied by using the laws of surds. To multiply or divide Surds they have to first be made of the same order.

Examples: $\sqrt{4}\times\sqrt{22}=\sqrt{88}$, $\sqrt{162}$ $\sqrt{18}=3\sqrt{2}$ / $\sqrt{9}=3\sqrt{2}$

Laws of Indices:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^{m/n} = \sqrt[n]{a^m}$
- $a^0 = 1$

Also, $\left(a^{\frac{1}{n}}\right)^n = a$

$$a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (ab)^{\frac{1}{n}} \quad \Rightarrow \quad (a^{\frac{1}{n}})^{\frac{1}{m}} = a^{\frac{1}{mn}}$$

Examples:

$$\sqrt[5]{4^3} = (4^3)^{\frac{1}{5}} = (4^{\frac{3}{5}})$$

$$5^3 \times 5^4 = 5^7 \quad \Rightarrow \quad \frac{5^5}{5^2} = 5^3$$

BASIC FORMULAS HELPFUL IN SIMPLIFICATION

$(a + b)^2 = a^2 + 2ab + b^2$

$(a^2 - b^2) = (a - b)(a + b)$

$(a + b)^2 = (a - b)^2 + 4ab$

$(a - b)^2 = (a + b)^2 - 4ab$

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = \left(x - \frac{1}{x}\right)^2 + 2$$

AVERAGE

Average is a general representation of a series. It is calculated by adding a given number of values and then dividing them the number of items. In statistics, average is referred as Arithmetic mean and is denoted by X.

$$\text{Hence, Average} = \frac{\text{Sum of elements}}{\text{Number of elements}} \text{ or } \frac{\sum X}{N}$$

Method to Solve Different Questions on Average

• Sum of elements = average × no. of elements

FORMULAS TO REMEMBER

• The average of first n natural numbers = $\frac{n+1}{2}$ • The average of the first n consecutive even numbers = $(n + 1)$	• The average of the first n consecutive odd numbers = n • The average of the first n odd natural numbers = $\left(\frac{\text{Last odd number}+1}{2}\right)$
• The average of the first n even numbers = $\left(\frac{\text{Last even number}+1}{2}\right)$ • The average of the squares of the first n natural numbers can be shown to be = $\frac{1}{6}(n + 1)(2n + 1)$ • The average of the squares of the first n even natural numbers = $\frac{2(n+1)(2n+1)}{3}$	• The average of the squares of the first n odd natural numbers = $\frac{(2n-1)(2n+1)}{3}$ • The average of the cubes of the first n natural numbers = $\frac{n(n+1)^2}{4}$

SIMPLE AND COMPOUND INTEREST

INTEREST

Basic terms associated with this topic:

Interest : It is the time value of money. It is the cost of using capital.

Principal : It is the borrowed amount.

Amount : It is the sum total of Interest and Principal.

Rate : It is the rate percent payable on the amount borrowed.
Period: It is the time for which the principal is borrowed.

Interest can be classified as: Simple interest : Simple Interest is payable on principal.

Compound Interest: Compound Interest is payable on Amount.

Basic formulas related to Simple Interest

- Simple Interest (SI) = $\frac{P \times R \times T}{100}$

  Here P = principal, R = rate per annum, T = time in years

  Amount (A) = $P + \frac{PRT}{100} = P\left(1 + \frac{RT}{100}\right)$ or P + SI

  If time is given in months, & Rate is given per annum, then

  $SI = \frac{P \times R \times T}{12 \times 100}$

  If time is given in weeks, & Rate is given per annum, then

  $SI = \frac{P \times R \times T}{52 \times 100}$

  If time is given in days, & Rate is given per annum, then

  $SI = \frac{P \times R \times T}{365 \times 100}$

- Also,

$$\text{Rate} = \frac{SI \times 100}{P \times T}$$

$$\text{Time} = \frac{SI \times 100}{P \times R}$$

$$\text{Principal} = \frac{SI \times 100}{R \times T}$$

If amount is given then,

$$\text{Principal} = \frac{Amt \times 100}{100 + (R \times T)}$$

PERCENTAGE

Percentage in mathematics, means to convert a given fraction to a denominator of 100. It is often denoted using the percent sign, "%". For example, 45% (read as "forty-five percent") is equal to 45%= 45/100= 0.45

Quicker Methods to Solve the Problems of Percent

• If price of a commodity is increased by x%, the consumption should be reduced, so that the expense remains the same, by

X/(100+x) × 100%

PROFIT AND LOSS

This chapter helps you to understand the intricacies of business world and the computation of profit or loss arising out of business transactions.

Various concepts related to this topic are :

Cost Price (CP): It is the price at which the item is procured by the seller.

Selling Price (SP): It is the price at which the item is sold by the seller.

Profit: It is the excess of the selling price over cost price, i.e.

Profit = SP – CP

Loss: It is the excess of cost price over the selling price, i.e.

Loss = CP – SP

Profit Percent: It is profit, expressed as a percentage of cost price, i.e.

$$\text{Profit Percent} = \frac{\text{Profit}}{\text{CP}} \times 100$$

Loss Percent: It is loss, expressed as a percentage of cost price, i.e,

$$\text{Loss Percent} = \frac{\text{Loss}}{\text{CP}} \times 100$$

Note: It should be kept in mind, that both profit and loss percent are calculated on cost price.

Formulas to ascertain cost price or selling price when profit or loss percent are given

To Find SP when Profit or Loss Percent & CP are given-

- In case Profit percent & CP is given

Then SP= $\left[\frac{100+\text{Profit\%}}{100}\right]\times \text{CP.}$

- In case loss percent & CP is given,

Then SP= $\left[\frac{100+\text{Profit\%}}{100}\right]\times \text{CP.}$

- When selling price and percentage profit are given, then

Cost price = selling price $\left(\frac{100}{100+\text{profit\%}}\right)$

To Find CP when profit or loss percent & SP are given-

- In case profit percent & SP is given,

$$\text{Then CP } = \left[\frac{100}{100+\text{Profit\%}}\right]\times \text{SP}$$

- In case loss percent & SP is given

$$\text{Then CP} = \left[\frac{100}{100 - \text{Loss\%}}\right] \times \text{SP}$$

Advanced Conditions:

• If two items are sold each at rupees R, one at a gain of x% and other at a loss of x %, there is always an overall loss given by $\frac{x^2}{100}$% and the value of loss is given by $\frac{2x^2R}{(100^2 - x^2)}$. In case the cost price of both the items is the same and percentage loss and gain are equal, then net loss or profit is zero.

TIME, SPEED AND DISTANCE

This chapter, deals with the following two types of questions:

(i) Time, Speed and Distance

(ii) Boat and Stream Time, Speed and Distance

Speed:

The distance covered per unit time is called speed. Speed is directly proportional to distance and inversely to time

• Distance = speed × time

- Speed = distance/ time
- Time = distance × speed

Conversion

- 1 km/hr $= \frac{5}{18}$ metre/second
- 1 metre/second $= \frac{18}{5}$ km/hr
- 1 Km/hr $= \frac{5}{8}$ mile/hr
- 1 mile/hr $= \frac{22}{15}$ foot/second

RELATIVE SPEED

If two trains are moving in opposite directions with a speed of X km/hr and Y km/hr respectively, then (X + Y) is their relative speed. In the other case if two trains are moving in the same direction with a speed of X km/hr and Y km/hr respectively, then (X – Y) is their relative speed.

For the first case the time taken by the trains in passing each other $= \frac{L_1 + L_2}{(X+Y)}$ hours.

where L1 and L2 are lengths of the trains.

For the second case the time taken by the trains in passing each other $= \frac{L_1 + L_2}{(X - Y)}$ hours

where L1 and L2 are lengths of the trains.

BOAT AND STREAM

When we move upstream, our speed gets deducted from the speed of the stream. Similarly when we move downstream our speed gets added to the speed of the stream.

Let the speed of a boat in still water be A km/hr and the speed of the stream (or current) be B km/hr, then

– Speed of boat with the stream i.e. speed downstream

= (A + B) km/hr

– Speed of boat against the stream i.e. speed upstream

= (A – B) km/hr

• **Boat's speed in still water**

$$= \frac{\text{speed downstream} + \text{speed upstream}}{2}$$

- **Speed of current**

$$= \frac{\text{Speed downstream} - \text{Speed upstream}}{2}$$

RATIO AND PROPORTION

Ratio:

Ratio gives us a relation between two quantities having similar unit. The ratio of A to B is written as A : B or A /B , where A is called the antecedent and B the consequent.

Proportion :

Proportion is an expression in which two ratios are equal.

For example $\frac{A}{B} = \frac{C}{D}$, $\Rightarrow A : B :: C : D$

Here, $A D = B C$

Properties of Ratio & Proportion :

- $a : b = m\,a : m\,b$, where m is a constant
- $a : b : c = A : B : C$ is equivalent to $\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$,

  This is an important property and used in the ratio of three quantities.
  If $a : b = c : d$, i.e

  $\frac{a}{b} = \frac{c}{d}$, then

  $\frac{b}{a} = \frac{d}{c}$, this is the property of Invertendo.
  If $a : b = c : d$, i.e

  $\frac{a}{b} = \frac{c}{d}$, then

  $\frac{a}{c} = \frac{b}{d}$, this is the property of Alternendo.
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{(a+b)}{b} = \frac{(c+d)}{d}$

  This property is called **Componendo**
  Also,
- $\frac{(a-b)}{b} = \frac{(c-d)}{d}$

This property is called Dividendo It also follows that:

- $\frac{(a-b)}{b} = \frac{(c-d)}{d}$

This property is called Dividendo It also follows that:

- $\frac{(a+b)}{(a-b)} = \frac{(c+d)}{(c-d)}$

This property is called Componendo and Dividendo

- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ldots$,Then

  $\frac{a+c+e+\cdots}{b+d+f+..}$ = each individual ratio i.e $\frac{a}{b}\, or\, \frac{c}{d}$

- If A > B then $\frac{A+C}{B+C} < \frac{A}{C}$

Where A, B and C are natural numbers In a proportion it should be remembered thatProduct of means = Product of extremes, i.e. b × c = a × d

Types of Proportion

Continued Proportion: We can say that a, b and c are in continued proportion, if

$$\frac{a}{b} = \frac{b}{c}$$
$$b^2 = ac \Rightarrow b = \sqrt{ac}$$

Here we can say that a is called first proportion, c is called third proportion and b is called mean proportion.

Also, if two nos. are given, and you are required to find mean proportion, then it should be written as

a : x :: x : b,

And is third proportion is to be computed, then it should be written as

a : b :: b : x.

Direct Proportion: If X is directly proportional to Y, that means any increase or decrease in any of two quantities will have proportionate effect on the other quantity. If X increases then Y will also increase and vice-versa.

Inverse Proportion: If X is inversely proportional to Y, that means any increase or decrease in any of two quantities will have inverse proportionate effect on the other quantity. This means if X increases, then Y decreases and if X decreases then Y increases and vice-versa for Y.

Applications of Ratio and Proportion

To find profit-sharing ratio on the basis of capital contribution.

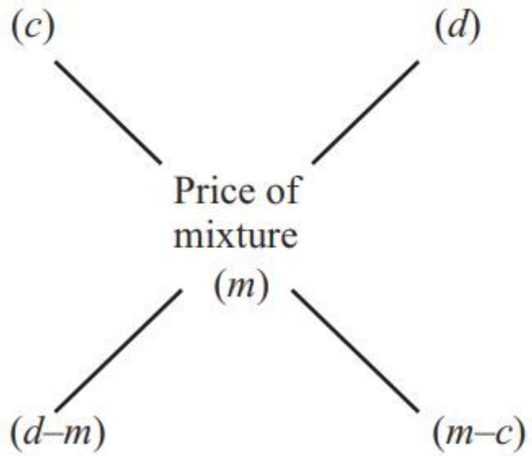
MIXTURES AND ALLIGATION

“Mixtures and alligations” is about mixing different qualities of goods in order to get desired levels/percentage/concentration of different objects.

Quicker Method to Solve Questions of Mixture and Alligation.
This rule helps us in solving questions where two varieties (of different prices) are mixed to get a new variety with a new Average price.

$$\frac{\text{Quantity of cheaper variety}}{\text{Quantity of dearer variety}} = \frac{\text{Price of Dearer variety} - \text{Average price}}{\text{Average price} - \text{Price of cheaper variety}}$$

$$\Rightarrow \frac{c}{d} = \frac{d-m}{m-c}$$



Then, (Cheaper quantity) : (dearer quantity)

$$= (d-m) : (m-c) \Rightarrow \frac{c}{d} = \frac{d-m}{m-c}$$

POINTS TO REMEMBER

- If in a partnership the investments made by first, second and third partners are x_1, x_2, x_3 respectively, the time period be t_1, t_2, t_3 then the ratio of profits is given by $x_1t_1 : x_2t_2 : x_3t_3$.

- If $x_1 : x_2 : x_3$ is the ratio of investments and $P_1 : P_2 : P_3$ be the ratio of Profit then time periods are given by $\frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3}$

• If $P_1 : P_2 : P_3$ is the ratio of profit on investments and $t_1 : t_2 : t_3$ be the ratio of time periods, then the ratio of investments will be

$$= \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$$

TIME AND WORK

In this chapter we will cover the following two topics they are based on similar concepts.

(i) Time and Work

(ii) Pipes and Cisterns

• Work is the job assigned or job completed. The rate of work is the speed or speed of work.

Quicker Method to solve the Questions of Work and Time

If a person completes a job in n days then he will complete 1/n th part in one day.

PIPES AND CISTERNS

Pipes and cisterns problems use the same principles as of time and work. Here a pipe connected with a cistern is called an inlet pipe to fill it or an outlet pipe to empty it.

Quicker Method to solve Questions on Pipes and Cisterns

• If an inlet pipe can fill a cistern in A hours, the part filled in

1 hour = $\frac{1}{A}$ (same as work and time fundamentals)

• If pipe A is ‘x’ times bigger than pipe B , then pipe A will

take $\frac{1}{x^{th}}$ of the time taken by pipe B to fill the cistern.

PERMUTATIONS AND COMBINATIONS PERMUTATION

Each of the arrangements, which can be made by taking, some or all of a number of things is called a Permutation. For Example : Formation of numbers, word formation, sitting arrangement in a row. The number of permutations of ‘n’ things taken ‘r’ at a time is

denoted by ${}^nP_r$. It is defind as, ${}^nP_r = \frac{n!}{(n-r)!}$.

Circular Permutations

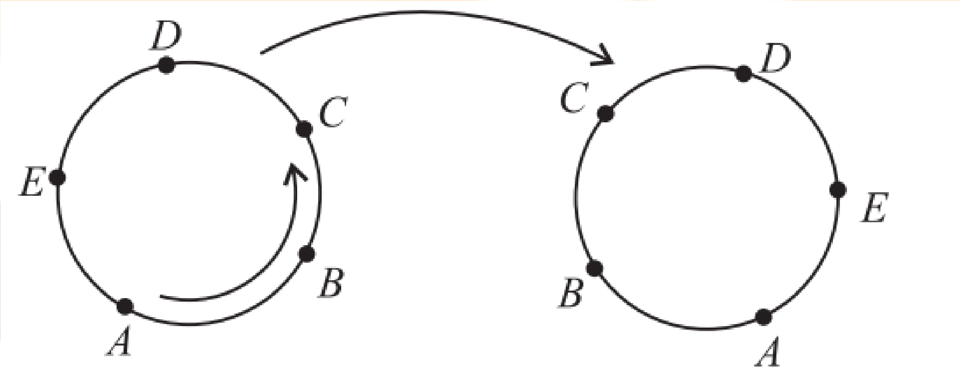
(i) Arrangements round a circular table :

The number of circular arrangement of n different things taken all at a time = (n – 1) !, if clockwise and anticlockwise orders are taken as different.

(ii) Arrangements of beads or flowers (all different) around a circular necklace or garland :

Consider five beads A, B, C, D and E in a necklace or five flowers A, B, C, D and E in a garland etc. If the necklace or garland on the left is turned over, we obtain the arrangement on the right, i.e., anticlockwise and clockwise order of arrangements are not different in the case of circular necklace and garland. Thus the number of circular permutations of 'n' different things taken. all at a time is $\frac{1}{2}(n-1)!$, if clockwise and

anticlockwise orders are taken to be same



Conditional Permutations

1. Number of permutations of n different things taken r at a time, in which a particular thing always occurs =r. $.^{n-1}P_{r-1}$.

2.Number of permutations of n different things taken r at a time, in which a particular thing never occurs = $^{n-1}P_r$.

3. Number of permutations of n different things taking all at a time, in which m specified things always come together = m! (n – m + 1)!.

4. Number of permutations of n different things taken all at a time, in which m specified things never come together = n! – m! (n – m + 1)!

COMBINATION

Each of the different selections that can be made from a given number of objects taken some or all of them at a time is called a Combination. The number of combinations of 'n' dissimilar things taken 'r' at a time is denoted by ^nC_r or C(n, r). It is defined as,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

POINTS TO REMEMBER

- $^nC_0 = 1, {}^nC_n = 1; {}^nP_r = r!\, {}^nC_r$
  $^nC_r = {}^nC_{n-r}$
- $^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
- $^nC_x = {}^nC_y \Rightarrow$ either $x = y$ or $x + y = n$
- $^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$
- $^nC_1 = {}^nC_{n-1} = n$

1. If there are n non-collinear points in a plane, then

(i) Number of straight lines formed = n_{C_2}

(ii) Number of triangles formed = n_{C_3}

(iii) Number of quadrilaterals formed = n_{C_4}

2. If there are n points in a plane out of which m are collinear, then

(i) Number of straight lines formed = $n_{C_2 - m_{C_2} + 1}$

(ii) Number of triangles formed = $n_{C_3 - m_{C_3} + 1}$

3. Number of diagonals of a polygon of n sides

is $^nC_2 - n$ *i.e.*, $\frac{n(n-3)}{2}$

4. Given n points on the circumference of a circle, then

(i) Number of straight lines obtained by joining these n points = n_{C_2}

(ii) Number of triangles obtained by joining these n points = n_{C_3}

(iii) Number of quadrilaterals obtained by joining these n points= n_{C_4}